

# Neutrino masses and $R$ -parity violation

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## Abstract

We review different contributions to the neutrino masses in the context of  $R$ -parity violating supersymmetry in a basis independent manner. We comment on the generic spectrum expected in such a scenario comparing different contributions.

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## 1 INTRODUCTION

In the last few years, neutrino oscillation experiments have established that neutrinos do have tiny masses[1, 2]. However, in the standard model (SM), neutrinos are massless. Hence these observations are evidences of beyond SM physics. The observed pattern of neutrino masses and mixing angles guides us in the search for new physics options beyond the SM.

Neutrino mass can be of Dirac type that conserves total lepton number and requires the presence of a right-handed neutrino, or it can be of Majorana type, that violates lepton number by two units. In the standard model, we do not have right-handed neutrinos and the lepton number is conserved. Hence neutrinos are massless. So SM needs to be augmented. However, to explain the smallness of neutrino masses, it is generally likely that an extension of the SM is lepton number violating. Supersymmetry offers a natural way out to accommodate lepton number violation and hence, non-zero neutrino masses.

Minimal supersymmetric standard model (MSSM) is an attractive candidate for physics beyond the standard model. It solves many theoretical puzzles posed by the SM and one expects to find its signatures in the forthcoming colliders. In the MSSM,  $R$ -parity conservation is imposed ad hoc on the Lagrangian to enforce lepton and baryon number conservations. As a result, like the SM, MSSM also fails to accommodate non-zero neutrino masses. However, Majorana neutrino masses can be generated if the requirement of  $R$ -parity conservation is relaxed. It is then interesting to explore how well  $R$ -parity violating supersymmetric models can explain the observed neutrino mass and mixing pattern.

Results from the recent neutrino oscillation experiments have more or less converged and a rough outline of the mixing pattern has emerged. It seems that it is quite different from our experience with the quark sector, where the mixing angles are very small. The data is best explained with the following set of parameters[3] that control neutrino oscillations:

$$\begin{aligned} \Delta m_{23}^2 &= 2.0 \times 10^{-3} \text{ eV}^2, & \Delta m_{12}^2 &= 7.2 \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{23} &= 0.5, & \sin^2 \theta_{12} &= 0.3, & \sin^2 \theta_{13} &< 0.074, \end{aligned} \quad (1)$$

where  $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$  are the squared mass differences and  $\theta_{ij}$  are the leptonic mixing angles. It suggests that there are one near-maximal ( $\theta_{23} \sim 45^\circ$ ), one large ( $\theta_{12} \sim 30^\circ$ ), and one rather small ( $\theta_{13} \leq 15^\circ$ ) mixing angles. However, neutrino oscillation experiments do not provide the overall mass scale. An upper limit can be obtained from the recent WMAP results[4] on the cosmic microwave background radiation, which imply that for three degenerate neutrinos, the common neutrino mass should be less than 0.23 eV. If we assume the lightest neutrino mass ( $m_1$ ) to be almost negligible, the data can be interpreted in a somewhat “hierarchical” mass scheme with  $m_3 \sim 0.04\text{eV}$  and  $m_2 \sim 0.008\text{eV}$ .

It is generally difficult to accommodate large mixing angles with large mass hierarchy in a theoretical model. It can be illustrated[1] with a two generation Majorana mass matrix

$$m_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}. \quad (2)$$

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It is clear that one can get a large mass hierarchy but not the large mixing when  $a \gg b, c$  or one can get large mixing without a hierarchy when  $a, b, c \sim 1$  and  $\det(m_\nu) \sim 1$ . But one can get large hierarchy with large mixing if  $a, b, c \sim 1$  and  $\det(m_\nu) \ll 1$ . However, in general, this is not expected naturally. An underlying mechanism is required to make the determinant small. In addition, a hierarchy in neutrino masses may be explained if different neutrinos get masses from different sources.  $R$ -parity violating ( $\mathcal{R}$ ) supersymmetric models are suitable in these regards. In these models, the heaviest neutrino gets mass at the tree level and the rest from the loops, thus generating a hierarchy.

In this short review, rather than attempting to present a comprehensive review of the entire literature[5], we will give a pedagogical introduction[6] to the different contributions to the neutrino masses in the MSSM with explicit  $R$ -parity violation. We will work in a generic scenario, where, in addition to the usual  $R$ -parity conserving couplings, all the lepton number violating couplings allowed by the symmetries are present. In the next section, we will describe our model and briefly discuss how to express the results in a basis independent framework. In section 4, we will discuss different tree and loop level diagrams contributing to the neutrino masses in detail. Their contributions will be cast in a basis independent notation and different suppression factors associated with them will be mentioned in section 5. We shall conclude in section 6.

## 2 $R$ -PARITY VIOLATION

In the SM, the field content and the requirement of renormalizability lead to global symmetries of the Lagrangian ensuring lepton number ( $L$ ) and baryon number ( $B$ ) conservations. In the MSSM, these accidental symmetries are no longer present.  $L$  and  $B$  conservations are guaranteed by imposing a discrete multiplicative symmetry called ‘ $R$ -parity’[7] on the Lagrangian. It is defined as  $R = (-1)^{3B+L+2S}$ , so that for all SM particles  $R = +1$  and for the super-particles  $R = -1$ .  $S$  denotes the spin of the particle. However in view of the observed proton stability, it is problematic to allow both  $L$  and  $B$  violating terms simultaneously. Conservation of  $B$  can be re-enforced by imposing a discrete baryon  $\mathbf{Z}_3$  symmetry[8, 9], thus allowing only  $L$  violating terms in the Lagrangian.

Once lepton number violation is allowed, there is no conserved quantum number that distinguishes the lepton supermultiplets  $\hat{L}_m$  ( $m = 1, 2, 3$ ) from the down-type Higgs supermultiplet  $\hat{H}_D$ . As a consequence, the down-type Higgs (Higgsino) mixes with the sleptons (leptons). It is therefore convenient to denote the four supermultiplets by one symbol  $\hat{L}_\alpha$  ( $\alpha = 0, 1, 2, 3$ ), with  $\hat{L}_0 \equiv \hat{H}_D$ . We use Greek indices to indicate the four dimensional extended lepton flavor space, and Latin ones for the usual three dimensional flavor space.

The most general renormalizable lepton number violating superpotential is given by[10]:

$$W = \epsilon_{ij} \left[ -\mu_\alpha \hat{L}_\alpha^i \hat{H}_U^j + \frac{1}{2} \lambda_{\alpha\beta m} \hat{L}_\alpha^i \hat{L}_\beta^j \hat{E}_m^c + \lambda'_{\alpha nm} \hat{L}_\alpha^i \hat{Q}_n^j \hat{D}_m^c - h_{nm}^U \hat{H}_U^i \hat{Q}_n^j \hat{U}_m^c \right], \quad (3)$$

where  $\hat{H}_U$  is the up-type Higgs supermultiplet;  $\hat{Q}_n$  is a doublet quark supermultiplet;  $\hat{U}_m$ ,  $\hat{D}_m$  and  $\hat{E}_m$  are singlet up-type quark, down-type quark and charged lepton supermultiplets respectively.  $\lambda_{\alpha\beta m}$  is anti-symmetric under the interchange of the indices  $\alpha$  and  $\beta$ . Note that the  $\mu$ -term of the MSSM ( $\mu_0$  in Eq. (3)) is now extended to a four-component vector  $\mu_\alpha$ . The down-type quark (charged lepton) Yukawa matrix of the MSSM, which corresponds to  $\lambda'_{0ij}$  ( $\lambda_{0ij}$ ) in Eq. (3), is now extended to  $\lambda'_{\alpha nm}$  ( $\lambda_{\alpha\beta m}$ ).

One needs to include the possible  $R$ -parity violating terms in the soft-supersymmetry-breaking sector as well. For example, one has to add  $R$ -parity violating  $A_{ijk}$ ,  $A'_{ijk}$  and  $B_i$  terms corresponding to the superpotential  $\mathcal{R}$  terms  $\lambda_{ijk}$ ,  $\lambda'_{ijk}$  and  $\mu_i$  respectively.  $\mathcal{R}$  scalar squared-mass terms also exist. The most general renormalizable  $R$ -parity violating soft-supersymmetry-breaking potential is given by[10]:

$$\begin{aligned} V_{\text{soft}} = & (M_Q^2)_{mn} \tilde{Q}_m^{i*} \tilde{Q}_n^i + (M_U^2)_{mn} \tilde{U}_m^* \tilde{U}_n + (M_D^2)_{mn} \tilde{D}_m^* \tilde{D}_n \\ & + (M_L^2)_{\alpha\beta} \tilde{L}_\alpha^{i*} \tilde{L}_\beta^i + (M_E^2)_{mn} \tilde{E}_m^* \tilde{E}_n + m_U^2 |H_U|^2 - (\epsilon_{ij} b_\alpha \tilde{L}_\alpha^i H_U^j + \text{h.c.}) \\ & + \epsilon_{ij} \left[ \frac{1}{2} A_{\alpha\beta m} \tilde{L}_\alpha^i \tilde{L}_\beta^j \tilde{E}_m + A'_{\alpha nm} \tilde{L}_\alpha^i \tilde{Q}_n^j \tilde{D}_m - A_{nm}^U H_U^i \tilde{Q}_n^j \tilde{U}_m + \text{h.c.} \right] \\ & + \frac{1}{2} \left[ M_3 \tilde{g} \tilde{g} + M_2 \tilde{W}^a \tilde{W}^a + M_1 \tilde{B} \tilde{B} + \text{h.c.} \right]. \end{aligned} \quad (4)$$

The  $B$ -term of the MSSM is now extended to  $B_\alpha$  in analogy with  $\mu_\alpha$  in the superpotential.  $A$  and  $A'$  terms are also extended in a similar fashion. The squared-mass term for the down-type Higgs boson and the  $3 \times 3$  slepton squared-mass matrix are combined and extended to the  $4 \times 4$  matrix  $(M_L^2)_{\alpha\beta}$ .

The total scalar potential comprises of contributions from the F-terms as calculated from the superpotential (3), the soft terms (4), and the D-terms. The expressions can be found in Ref. [10]. The contribution of the neutral scalar fields to the scalar potential is given by,

$$V_{\text{neutral}} = (m_U^2 + |\mu|^2) |H_U|^2 + [(M_L^2)_{\alpha\beta} + \mu_\alpha \mu_\beta^*] \tilde{\nu}_\alpha \tilde{\nu}_\beta^* - (B_\alpha \tilde{\nu}_\alpha H_U + B_\alpha^* \tilde{\nu}_\alpha^* H_U^*) + \frac{1}{8}(g^2 + g'^2) [|H_U|^2 - |\tilde{\nu}_\alpha|^2]^2, \quad (5)$$

where  $|\mu|^2 \equiv \sum_\alpha |\mu_\alpha|^2$ . Note that to simplify notation, here we denote the neutral components of the up-type scalar doublets by  $H_U$ . The vacuum expectation values (VEV) for the neutral scalars denoted by  $\langle H_U \rangle \equiv \frac{v_u}{\sqrt{2}}$  and  $\langle \tilde{\nu}_\alpha \rangle \equiv \frac{v_\alpha}{\sqrt{2}}$  are determined by the following minimization conditions which follow from Eq. (5):

$$(m_U^2 + |\mu|^2) v_u^* = B_\alpha v_\alpha - \frac{1}{8}(g^2 + g'^2)(|v_u|^2 - |v_d|^2) v_u^*, \quad (6)$$

$$[(M_L^2)_{\alpha\beta} + \mu_\alpha \mu_\beta^*] v_\beta^* = B_\alpha v_u + \frac{1}{8}(g^2 + g'^2)(|v_u|^2 - |v_d|^2) v_\alpha^*, \quad (7)$$

with  $|v_d|^2 \equiv \sum_\alpha |v_\alpha|^2$ . We further define,

$$v \equiv (|v_u|^2 + |v_d|^2)^{1/2} = \frac{2 m_W}{g} \simeq 246 \text{ GeV}, \quad \tan \beta \equiv \frac{v_u}{v_d}. \quad (8)$$

For simplicity, we will assume that the gaugino mass parameters  $M_i$ ,  $(M_L^2)_{\alpha\beta}$ ,  $\mu_\alpha$ ,  $B_\alpha$  and  $v_\alpha$  are real and choose  $\tan \beta$  to be positive.

Treating  $\hat{H}_d$  and  $\hat{L}_i$  in the same footing entails a freedom in choosing the direction of Higgs in the four dimensional extended flavor space, which results from the absence of an unique interaction eigenstate basis. However, talking in terms of the numerical sizes of these  $\mathcal{R}$  couplings in the context of some phenomenological consideration makes sense only if one specifies the basis. So one can either choose some specific basis; for example, one can rotate to a basis where  $\mu_i = 0$  or  $v_i = 0$ . The other option[11, 12, 13] is to express experimental observables in terms of basis invariant quantities, which we outline in the following section.

### 3 Basis Independence

In the four dimensional vector space spanned by  $(\hat{H}_d, \hat{L}_1, \hat{L}_2, \hat{L}_3)$ , one can construct<sup>1</sup> coupling constant combinations invariant under rotations in this space. It is interesting to note that it is convenient to choose a basis while defining basis invariants. As a first step one should pick up a Higgs direction. Although any direction can be identified as the Higgs direction, the coupling constants offer many directions to choose from. One can identify  $\mu_\alpha$  as the direction for the Higgs, for which  $\mu_i = 0$ . Similarly,  $v_\alpha$ ,  $\lambda'_{\alpha nm} [\equiv (\lambda'_{nm})_\alpha]$  are vectors in this space and can very well be chosen as Higgs directions.  $\lambda_{\alpha\beta p}$  are anti-symmetric matrices and it helps us to choose the lepton directions orthogonal to the Higgs direction, thus defining the basis. An example will help to clarify the mechanism.

Let us choose the Higgs direction to be  $v_\alpha/v_d$ , which makes sneutrino VEV's to vanish. The lepton directions are taken as  $v^\alpha \lambda_{\alpha\beta p}/|v^\alpha \lambda_{\alpha\beta p}|$ . The anti-symmetry in  $\lambda_{\alpha\beta p}$  ensures that the lepton directions are orthogonal to the Higgs direction. The orthogonality amongst the lepton directions is guaranteed by choosing the right-handed lepton basis, such that  $v^\alpha \lambda_{\alpha\gamma p} \lambda_{\beta\gamma q} v^\beta \propto \delta_{pq}$ . If  $R$ -parity is conserved, this amounts to choosing the charged lepton mass eigenstate basis. Similarly the left-handed quark and right-handed down-type quark bases are chosen such that the down-type quark mass matrix (proportional to  $v^\alpha \lambda'_{\alpha pq}$ ) is diagonal. We have neglected in this discussion the mixing between the charged leptons and the

<sup>1</sup>Here we will follow Ref. [11]. For an alternative approach see Ref. [13].

charginos, induced by the  $R$ -parity violating bilinear  $\mu_i$  parameters, which are expected to be small due to the observed smallness of the neutrino masses. These effects are included as perturbations – through “mass insertions” (explained afterwards) in the Feynman diagram calculations, so that the results are basis independent up to the order of square of the  $\mathcal{R}$  parameters. The deviation of the charged lepton basis from its mass eigenstate basis due to the presence of the small  $\mathcal{R}$  parameters will show up only at higher orders.

We are now ready to write down the invariants with respect to the basis we chose in the last paragraph. First we note that any scalar product constructed out of the aforesaid vectors and matrices in this four dimensional vector space are invariants. But we should choose those which will be useful to express different contributions to neutrino mass discussed in the next section. One such a set is given by[14]:

$$\begin{aligned}\delta_i^\mu &= \frac{\mu^\beta}{|\mu|} \frac{v^\alpha \lambda_{\alpha\beta i}}{|v^\alpha \lambda_{\alpha\beta i}|}, & \delta_i^B &= \frac{B^\beta}{|B|} \frac{v^\alpha \lambda_{\alpha\beta i}}{|v^\alpha \lambda_{\alpha\beta i}|}, \\ \delta_{ipq}^{\lambda'} &= \lambda'_{\beta pq} \frac{v^\alpha \lambda_{\alpha\beta i}}{|v^\alpha \lambda_{\alpha\beta i}|}, & \delta_{ijk}^\lambda &= \frac{v^\alpha \lambda_{\alpha\beta i}}{|v^\alpha \lambda_{\alpha\beta i}|} \lambda_{\beta\gamma k} \frac{v^\theta \lambda_{\theta\gamma j}}{|v^\theta \lambda_{\theta\gamma j}|}.\end{aligned}\tag{9}$$

$\delta_i^\mu$ 's correspond to the projections of the vector  $\mu_\alpha/|\mu|$  onto the lepton directions. Similar geometric interpretations can be made for the other invariants as well[11]. These dimensionless invariants have the property that they take simple forms in the vanishing sneutrino VEV basis,  $v_i = 0$ :

$$\delta_i^\mu = \frac{\mu_i}{|\mu|}, \quad \delta_i^B = \frac{B_i}{|B|}, \quad \delta_{ijk}^{\lambda'} = \lambda'_{ijk}, \quad \delta_{ipq}^\lambda = \lambda_{ipq}.\tag{10}$$

As the name suggests, the numerical values of these invariants remain the same whichever basis one chooses, although their analytical forms might change. In particular, they are zero if there is no  $R$ -parity violation. Although these invariants were constructed choosing a specific Higgs direction, one does not need to refer to any specific basis while calculating experimental observables in terms of these invariants, which is a must if one uses the usual coupling constants instead of these invariant combinations. However, the simple forms noted in Eq. (10) suggest that one can proceed as follows, rather than working in a general basis:

- Work in the  $v_i = 0$  basis,
- calculate the neutrino mass contributions in terms of the  $\mathcal{R}$  parameters  $\mu_i$ ,  $B_i$ ,  $\lambda_{ipq}$ , and  $\lambda'_{ijk}$ ,
- and then replace these with the basis invariant parameters  $\delta_i^\mu$ ,  $\delta_i^B$ ,  $\delta_{ipq}^\lambda$ , and  $\delta_{ijk}^{\lambda'}$  respectively, using Eq. (10).

This is equivalent to working in a general basis but is much simpler. Thus one can get basis invariant results although in the intermediate steps one can afford to work in a specific basis. Here we will generally work in  $v_i = 0$  basis. As we will see in the next section that tree level neutrino contribution is unacceptably large unless the vectors  $v_\alpha$  and  $\mu_\alpha$  are nearly parallel, we expect that  $\mu_i$  will be very small in this basis. This also means that we will be very close to the charged lepton mass eigenstate basis. The fact that the  $\mathcal{R}$  couplings are small in this basis will also enable us to work in the “mass insertion approximation” which means for propagating fields in the diagrams we will be using the  $R$ -parity conserving usual MSSM mass eigenstates and include the  $R$ -parity violating parameters as insertions in the diagram. Smaller  $\mathcal{R}$  parameters would ensure better accuracy as we make this approximation in the calculation of the Feynman diagrams in the next section.

## 4 CONTRIBUTIONS TO THE NEUTRINO MASSES

$R$ -parity violating MSSM allows terms in the Lagrangian which violates lepton number by one unit. Now two of these can be taken together to construct a neutrino Majorana mass term which violates lepton number by two units. Contributions to neutrino masses can come from both tree and loop level diagrams

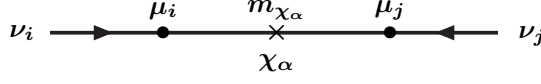


Figure 1: Tree level neutrino mass in the mass insertion approximation. A blob represents mixing between the neutrino and the up-type Higgsino. The cross on the neutralino propagator signifies a Majorana mass term for the neutralino.

and an extensive literature exists on this subject[15]. The one-loop contributions from the trilinear  $\mathcal{R}$  couplings were much discussed, until it was realized[16, 17] that sneutrino–anti-sneutrino mixing can play a significant role in generating neutrino masses at one-loop. In Ref. [14] many more loops were identified. These loops were also studied in Refs. [18, 19, 20]. In what follows, we will discuss only the dominant contributions in a generic scenario.

#### 4.1 TREE LEVEL CONTRIBUTION

As mentioned earlier, the neutrinos mix with neutralinos as  $R$ -parity is violated. Hence, at the tree level, the neutrino mass matrix receives contributions as shown in Fig. 1. In other words, the usual  $4 \times 4$  neutralino mass matrix in MSSM gets extended to a  $7 \times 7$  neutralino–neutrino mass matrix. In  $\{\tilde{B}, \tilde{W}_3, \tilde{H}_U, \tilde{H}_D, \nu_1, \nu_2, \nu_3\}$  basis, it is given by[21, 22, 10]:

$$M^{(n)} = \begin{pmatrix} M_1 & 0 & m_Z s_W \frac{v_u}{v} & -m_Z s_W \frac{v_d}{v} & 0 & 0 & 0 \\ 0 & M_2 & -m_Z c_W \frac{v_u}{v} & m_Z c_W \frac{v_d}{v} & 0 & 0 & 0 \\ m_Z s_W \frac{v_u}{v} & -m_Z c_W \frac{v_u}{v} & 0 & \mu & \mu_1 & \mu_2 & \mu_3 \\ -m_Z s_W \frac{v_d}{v} & m_Z c_W \frac{v_d}{v} & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

where  $c_W \equiv \cos \theta_W$  and  $s_W \equiv \sin \theta_W$ . This is a rank 5 matrix and taking into account all the four non-zero neutralino masses, we see that only one neutrino gets massive at the tree level. The four neutralinos can be integrated out to find the neutrino mass matrix

$$[m_\nu]_{ij}^{(\mu\mu)} = C \mu_i \mu_j, \quad (12)$$

where,

$$C = \frac{m_Z^2 m_{\tilde{\gamma}} \cos^2 \beta}{\mu(m_Z^2 m_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu)} \sim \frac{\cos^2 \beta}{\tilde{m}}, \quad (13)$$

such that  $m_{\tilde{\gamma}} \equiv c_W^2 M_1 + s_W^2 M_2$  and in the last step we assume that all the relevant masses are at the electroweak (or supersymmetry breaking) scale,  $\tilde{m}$ . The only non-zero eigenvalue of  $[m_\nu]_{ij}^{(\mu\mu)}$  is given by,

$$m_3 = C (\mu_1^2 + \mu_2^2 + \mu_3^2). \quad (14)$$

This is the mass of the only neutrino which gets massive at the tree level. We see that it is proportional to the  $R$ -parity violating quantity  $\sum \mu_i^2$  and to  $\cos^2 \beta$ . For large  $\tan \beta$  the latter is a suppression factor.

For the following discussion, it is convenient to cast Eq. (14) in a basis-invariant form

$$m_3 = \frac{m_Z^2 \mu m_{\tilde{\gamma}} \cos^2 \beta \sin^2 \xi}{m_Z^2 m_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu}, \quad (15)$$

where  $\xi$  is a measure of the misalignment of  $v_\alpha$  and  $\mu_\alpha$ :

$$\cos \xi = \frac{\sum_\alpha v_\alpha \mu_\alpha}{v_d \mu}. \quad (16)$$

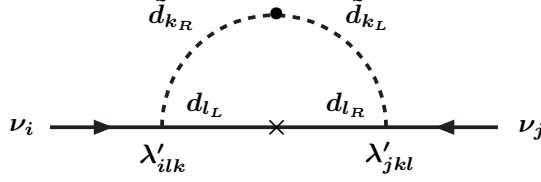


Figure 2: Trilinear loop contribution to the neutrino mass matrix. The blob on the scalar line indicates mixing between the left-handed and the right-handed squarks. A mass insertion on the internal quark propagator is denoted by the cross.

In a generic supersymmetric model one does not expect any alignment between  $v_\alpha$  and  $\mu_\alpha$  and this leads to an unacceptably large neutrino mass. This poses a problem for this kind of models as fine-tuning is needed to achieve an exact or approximate alignment. Moreover, this is not renormalization group invariant. That is, if we achieve an alignment at some particular scale by fine-tuning our model parameters, at some other scale a misalignment is expected to pop up due to running of the parameters.

However, if some underlying mechanism ensures  $v_\alpha \propto \mu_\alpha$ , the predicted mass can be reconciled with experimental observations. Here the following observation plays a significant role.

Sufficient conditions<sup>2</sup> to achieve the required alignment  $v_\alpha \propto \mu_\alpha$  are:

- $B_\alpha$  is aligned with  $\mu_\alpha$ :

$$B_\alpha \propto \mu_\alpha, \quad (17)$$

- $\mu_\alpha$  is an eigenvector of  $(M_L^2)_{\alpha\beta}$ :

$$(M_L^2)_{\alpha\beta} \mu_\beta = \tilde{m}^2 \mu_\alpha. \quad (18)$$

To show this, it is convenient to go to a basis in which  $(M_L^2)_{\alpha\beta}$  is diagonal and consequently Eq. (18) implies that in this basis  $\mu_\alpha$  has only one non-zero component, say  $\mu_0$ . Then Eq. (17) ensures that for  $B_\alpha$ , only  $B_0$  is non-zero. Then, as in the  $R$ -parity conserving case, from the minimum equations it follows trivially that  $v_\alpha$  has the only non-zero component  $v_0$ . This implies  $v_\alpha \propto \mu_\alpha$ .

The required alignment can be ensured by approximately satisfying Eqs. (17) and (18) in the framework of horizontal symmetries[21]. Another approach is to consider high scale alignment models. In these models, universality conditions at some unification scale are assumed in a way to ensure that Eqs. (17) and (18) are satisfied exactly at that scale. Then as one runs down the energy scale, the parameters  $(M_L^2)_{\alpha\beta}$  and  $B_\alpha$  evolve, generating a misalignment[22] between  $v_\alpha$  and  $\mu_\alpha$ . This running is governed by charged lepton or down-type quark Yukawa couplings and to have a misalignment it is necessary to have a non-zero  $L$ -violating  $\lambda$  or  $\lambda'$  coupling or a corresponding  $A$  term. If there are no suppressions coming from these  $L$ -violating terms, the neutrino mass is at least 1 MeV (1 GeV)[22] if the unification scale is taken to be of the order of  $M_{\text{Planck}} \sim 10^{19}$  GeV ( $10^6$  GeV). Hence one needs a further strong suppression from these  $L$ -violating terms to get a realistic neutrino mass.

We conclude this section noting that in a generic supersymmetric model with  $R$ -parity violation, one can not avoid one neutrino being massive at the tree level.

## 4.2 TRILINEAR ( $\lambda\lambda$ AND $\lambda'\lambda'$ ) LOOPS

Neutrino masses receive contribution from fermion-sfermion loops (see Fig. 2) dependent on the trilinear  $R$ -parity violating couplings  $\lambda$  and  $\lambda'$ . These types of loops have attracted much attention in the literature. Here we only present approximated expressions which are sufficient for our study. Complete expressions can be found, for example, in Ref. [10].

<sup>2</sup>For a more general condition see Ref. [10].

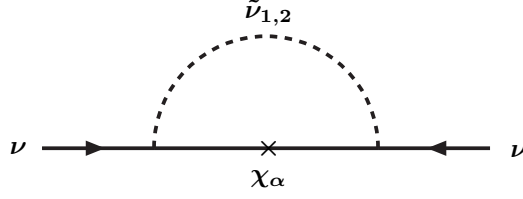


Figure 3: Majorana neutrino mass induced by sneutrino mass splitting. The cross on the internal neutralino propagator denotes a Majorana mass for the neutralino.

Neglecting quark flavor mixing, the contribution of the  $\lambda'\lambda'$  loops is proportional to the internal down-type quark mass and to the mixing between left and right down-type squarks. Explicitly,

$$[m_\nu]_{ij}^{(\lambda'\lambda')} \approx \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk} \lambda'_{jkl} \frac{m_{d_l} \Delta m_{\tilde{d}_k}^2}{m_{\tilde{d}_k}^2} \sim \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk} \lambda'_{jkl} \frac{m_{d_l} m_{d_k}}{\tilde{m}}, \quad (19)$$

where  $m_{\tilde{d}_k}$  is the average  $k$ th squark mass,  $\Delta m_{\tilde{d}_k}^2$  is the squared mass splitting between the two  $k$ th squarks, and in the last step we used  $\Delta m_{\tilde{d}_k}^2 \approx m_{d_k} \tilde{m}$  and  $m_{\tilde{d}_k} \sim \tilde{m}$ . In the numerator 3 is the color factor. There are similar contributions from loops with intermediate charged leptons and sleptons where  $\lambda'$  is replaced by  $\lambda$  and there is no color factor in the numerator:

$$[m_\nu]_{ij}^{(\lambda\lambda)} \approx \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk} \lambda_{jkl} \frac{m_{\ell_l} \Delta m_{\tilde{\ell}_k}^2}{m_{\tilde{\ell}_k}^2} \sim \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk} \lambda_{jkl} \frac{m_{\ell_l} m_{\ell_k}}{\tilde{m}}. \quad (20)$$

We see that the trilinear loop-generated masses are suppressed by the  $R$ -parity violating parameters  $\lambda'^2$  ( $\lambda^2$ ), by a loop factor, and by two down-type quark (charged lepton) masses. The latter factor, absent in other types of loops, makes the trilinear contribution irrelevant in most cases.

### 4.3 BB LOOPS

In the presence of bilinear  $\mathcal{R}$  terms, the sneutrinos and the neutral Higgses mix at the tree level, which mixes sneutrinos and anti-sneutrinos inducing a splitting between sneutrino mass eigenstates. This generates a Majorana neutrino mass at one-loop. We explain this mechanism in more detail in the following.

For simplicity let us work with one fermion generation. We assume CP conservation. This enables us to consider CP even and CP odd scalar sectors separately[10, 23]. The CP even sector consists of: the light neutral Higgs  $h$ , the heavier neutral Higgs  $H$ , the CP even sneutrino  $\tilde{\nu}_+$ . The CP odd scalar sector consists of: the pseudoscalar Higgs  $A$ , the Goldstone boson  $G$  (corresponding to the  $Z$ ), the CP odd sneutrino  $\tilde{\nu}_-$ . With  $R$ -parity conservation,  $\tilde{\nu}_\pm$  do not mix with the Higgses and are degenerate. So one defines the eigenstates of lepton number:  $\tilde{\nu} \equiv (\tilde{\nu}_+ + i\tilde{\nu}_-)/\sqrt{2}$  and  $\tilde{\bar{\nu}} \equiv (\tilde{\nu}_+ - i\tilde{\nu}_-)/\sqrt{2}$ . The sneutrino mass squared term ( $\Delta L = 0$ ) is given by  $m_{\tilde{\nu}}^2 \tilde{\nu}^* \tilde{\nu}$ .

When  $R$ -parity is violated, a  $\Delta L = 2$  mass term  $m_{\Delta L=2}^2 \tilde{\nu} \tilde{\nu}$  gets generated which mixes the sneutrino and the anti-sneutrino inducing a splitting between the sneutrino mass eigenstates  $\tilde{\nu}_{1,2}$ . We assume the  $\mathcal{R}$  couplings to be small which allows us to work perturbatively. Now  $\tilde{\nu}_+$  mixes with  $h$  and  $H$  and  $\tilde{\nu}_-$  mixes with  $A$  and  $G$  and these mixings are proportional to  $B_1$ . As a result, as the  $3 \times 3$  squared mass matrices in each CP sector gets diagonalized, the sneutrino in each sector experiences a shift (to the leading order in  $B_1$ ) in their squared mass eigenvalues proportional to  $B_1^2$ . Now, the two shifts are not equal, leaving behind a splitting between  $\tilde{\nu}_{1,2}$ :  $\Delta m_{\tilde{\nu}} \sim B_1^2/\tilde{m}^3$ . It induces a Majorana neutrino mass proportional to  $\Delta m_{\tilde{\nu}}$  at one-loop (see Fig. 3).

For quantitative estimations, we will recast the same diagram in the “mass insertion approximation” as shown in Fig. 4. The contribution from this diagram is proportional to two insertions of  $B_i$  parameters. However, it should be mentioned that in addition to the  $B_i$  terms, the terms like  $(M_L^2)_{0i}$  and  $\mu_i$  also contribute (see Eq. (5)). However, these contributions are related to  $B_i$  through the minimum equation

(Eq. (7)) in the  $v_i = 0$  basis, namely  $[(M_L^2)_{0i} + \mu_0 \mu_i] = -\tan \beta B_i$ . So we will designate these all-bilinear loops as  $BB$  loops.

The contribution to the neutrino mass matrix from  $BB$  loop is given by[14],

$$[m_\nu]_{ij}^{(BB)} = \sum_\alpha \frac{g^2 B_i B_j}{4 \cos^2 \beta} (Z_{\alpha 2} - Z_{\alpha 1} g' / g)^2 m_{\chi_\alpha} \{ I_4(m_h, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \cos^2(\alpha - \beta) + I_4(m_H, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \sin^2(\alpha - \beta) - I_4(m_A, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha}) \} , \quad (21)$$

where  $Z_{\alpha\beta}$  is the neutralino mixing matrix with  $\alpha, \beta = 1, \dots, 4$  and

$$\begin{aligned} I_4(m_1, m_2, m_3, m_4) &= \frac{1}{m_1^2 - m_2^2} [I_3(m_1, m_3, m_4) - I_3(m_2, m_3, m_4)] , \\ I_3(m_1, m_2, m_3) &= \frac{1}{m_1^2 - m_2^2} [I_2(m_1, m_3) - I_2(m_2, m_3)] , \\ I_2(m_1, m_2) &= -\frac{1}{16\pi^2} \frac{m_1^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} . \end{aligned} \quad (22)$$

Assuming that all the masses in the right-hand side of Eq. (21) are of the order of the weak scale, we estimate

$$[m_\nu]_{ij}^{(BB)} \sim \frac{g^2}{64\pi^2} \frac{1}{\cos^2 \beta} \frac{B_i B_j}{\tilde{m}^3} \epsilon_H \quad (23)$$

where,

$$\epsilon_H \equiv \left| \frac{I(m_h) \cos^2(\alpha - \beta) + I(m_H) \sin^2(\alpha - \beta) - I(m_A)}{|I(m_h)| \cos^2(\alpha - \beta) + |I(m_H)| \sin^2(\alpha - \beta) + |I(m_A)|} \right| , \quad (24)$$

where  $I(x) \equiv I_4(x, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}, m_{\chi_\alpha})$  [see Eq. (21)], and the  $i, j, \alpha$  indices of  $\epsilon_H$  are implicit.

Naively, one would expect  $\epsilon_H \sim 1$ . However, it has been pointed out in Ref. [20] that it is parameter space dependent and can provide a suppression of several orders of magnitude. This suppression is related to the Higgs decoupling which is typical[24] to the two Higgs doublet models like MSSM. At the decoupling limit,  $\cos^2(\alpha - \beta) \rightarrow 0$ ,  $m_H \simeq m_A \gg m_h \simeq m_Z$ , we see that  $\epsilon_H \rightarrow 0$ . The three Higgs loops tend to cancel each other, which becomes more and more severe as we approach the decoupling limit. This can be further clarified if we look at the weighted sum of the three Higgs propagators before integrating over the internal momenta  $k$ :

$$P_S = \frac{1}{k^2 - m_h^2} \cos^2(\alpha - \beta) + \frac{1}{k^2 - m_H^2} \sin^2(\alpha - \beta) - \frac{1}{k^2 - m_A^2} . \quad (25)$$

For simplicity we use the tree level relations[25, 26]

$$\cos^2(\alpha - \beta) = \frac{m_h^2(m_Z^2 - m_h^2)}{m_A^2(m_H^2 - m_h^2)}, \quad m_Z^2 - m_h^2 = m_H^2 - m_A^2, \quad (26)$$

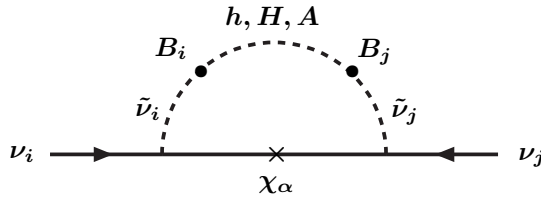


Figure 4: The  $BB$  loop-generated neutrino mass. This is the same diagram as in Fig. 3, but re-drawn in the mass insertion approximation. Here the blobs denote mixing of the sneutrinos with the neutral Higgs bosons.



to obtain

$$P_S = \frac{-k^2(m_Z^2 - m_h^2)(m_A^2 - m_h^2)}{m_A^2(k^2 - m_H^2)(k^2 - m_A^2)(k^2 - m_h^2)}. \quad (27)$$

In the approach to the decoupling limit,  $P_S$  scales inversely to the fourth power of the heavy mass. Thus the weighted sum falls faster than the individual propagators contributing to  $P_S$ , indicating an increasingly stronger cancellation between the Higgs diagrams. As one includes loop corrections, modifying the tree level relations Eq. (26), some cancellation still occurs. The point to note is that a partial cancellation occurs even away from the decoupling limit as the  $I_4$  functions do not change much with respect to its arguments. This makes  $\epsilon_H$  to emerge as a suppression factor.

Another interesting point is that this Higgs cancellation becomes more significant for high  $\tan\beta$ . This significantly reduces the enhancement effect from the  $1/\cos^2\beta$  factor in Eq. (21) for high  $\tan\beta$ , making  $[m_\nu]_{ij}^{(BB)}$  less sensitive to  $\tan\beta$ .

Next we study the  $BB$  loop effect on the neutrino masses. For this we rewrite Eq. (21) as

$$[m_\nu]_{ij}^{(BB)} = C_{ij} B_i B_j. \quad (28)$$

Now if sneutrinos of all generations are degenerate, all  $C_{ij}$ 's will be equal and only one neutrino will be massive from the  $BB$  loops. In general one does not expect  $B_\alpha \propto \mu_\alpha$ . So the neutrino which gets mass from the  $BB$  loops is not the same one which got massive at the tree level (see Eq. (12)). Hence if we consider only the tree level and  $BB$  loop contributions, in the case of degenerate sneutrinos one neutrino is left massless. Although it can get mass from other loops, it is interesting to note that it can be massive from the  $BB$  loops itself if the sneutrinos are non-degenerate<sup>3</sup>, making  $C_{ij}$  no longer a rank one matrix. Hence one would expect that the contribution to the lightest neutrino mass from  $BB$  loops will be dependent on the amount of non-degeneracy in the sneutrino sector. In what follows, we will try to quantify this dependence when the non-degeneracy is small compared to the sneutrino masses.

We assume that the heaviest neutrino acquires large mass at the tree level. This helps us to deal only with the loop contributions to the first two generations, simplifying our problem.

We take the mass-squares of the two sneutrinos as  $(m_\nu^2)_{1,2} \equiv m_\nu^2(1 \pm \Delta)$ . After we plug them in Eq. (21) and diagonalize, we get the following mass eigenvalues in their leading order:

$$m_2 = (B_1^2 + B_2^2) f_1, \quad m_1 = \frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (4f_1 f_3 - f_2^2) \Delta^2, \quad (29)$$

where  $f_i$ 's are the coefficients[20] of the loop function expanded in powers of  $\Delta$ . We see that  $m_2$  is the same as that in the degenerate case.  $m_1$  is proportional to the square of the sneutrino mass splitting between generations.

We define the following measure of the degeneracy suppression

$$\epsilon_D \equiv \frac{m_1}{m_2} = f_c \frac{B_1^2 B_2^2}{(B_1^2 + B_2^2)^2} \Delta^2, \quad f_c = \frac{4f_1 f_3 - f_2^2}{f_1}. \quad (30)$$

We note that, in addition to the  $\Delta^2$  suppression, the lightest neutrino mass is also suppressed by  $f_c \sim 0.1$ [20].

## 4.4 $\mu B$ LOOPS

There is another type of diagram (Fig. 5), which involves both neutrino-Higgsino mixing (through  $\mu_i$  terms) and sneutrino-Higgs mixing (through  $B_i$  terms). The contribution from this diagram to the neutrino mass matrix is given by[14, 20]<sup>4</sup>,

$$[m_\nu]_{ij}^{(\mu B)} = \sum_{\alpha, \beta} \frac{g^2}{4 \cos \beta} \mu_i B_j \frac{m_{\chi_\beta}}{m_{\chi_\alpha}} Z_{\alpha 3} (Z_{\beta 2} - Z_{\beta 1} g' / g)$$

<sup>3</sup>It should be emphasized that here we are talking about non-degeneracy of sneutrinos of different generations, which is different from the non-degeneracy of the sneutrino mass eigenstates of a given generation induced by the bilinear  $R$ -parity violating parameters.

<sup>4</sup>Note that in this expression, there is a typographical error in sign in Ref. [14], which has been corrected in Ref. [20].

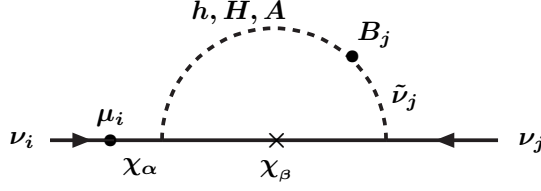


Figure 5: Neutrino Majorana mass generated by  $\mu B$  loop. The cross on the internal neutralino line denotes, as before, a Majorana mass term for the neutralino. There exists another diagram with  $i \leftrightarrow j$ .

$$\begin{aligned}
& \left\{ - \left[ Z_{\alpha 4} (Z_{\beta 2} - Z_{\beta 1} g' / g) \sin \alpha + (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 3} \cos \alpha \right. \right. \\
& \quad \left. \left. + (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 4} \sin \alpha \right] \cos(\alpha - \beta) I_3(m_h, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \right. \\
& \quad + \left[ Z_{\alpha 4} (Z_{\beta 2} - Z_{\beta 1} g' / g) \cos \alpha - (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 3} \sin \alpha \right. \\
& \quad \left. + (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 4} \cos \alpha \right] \sin(\alpha - \beta) I_3(m_H, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \\
& \quad + \left[ Z_{\alpha 4} (Z_{\beta 2} - Z_{\beta 1} g' / g) \sin \beta + (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 3} \cos \beta \right. \\
& \quad \left. + (Z_{\alpha 2} - Z_{\alpha 1} g' / g) Z_{\beta 4} \sin \beta \right] I_3(m_A, m_{\chi_\beta}, m_{\tilde{\nu}_j}) \Big\} + (i \leftrightarrow j) \quad (31)
\end{aligned}$$

Taking all the weak scale masses as  $\tilde{m}$ , these contributions to the neutrino mass matrix are given approximately by

$$[m_\nu]_{ij}^{(\mu B)} \sim \frac{g^2}{64\pi^2} \frac{1}{\cos \beta} \frac{\mu_i B_j + \mu_j B_i}{\tilde{m}^2} \epsilon'_H. \quad (32)$$

Here  $\epsilon'_H$  is a similar suppression factor expected from cancellation between different Higgs diagrams as in the case of  $BB$  loops.

Comparing Eq. (32) with Eq. (23), we see that in the flavor basis these diagrams are expected to yield similar contributions to the  $BB$  loops. But due to the dependence on  $\mu_i$ , the  $\mu B$  loop contribution to the neutrino masses is sub-leading[27, 20], if the tree level contribution is the dominant one. For illustration, let us consider a two generation case and consider contributions from the tree level and  $\mu B$  diagrams only. Now a look at Eq. (31) reveals that one can recast it in the following form[20]:

$$[m_\nu]_{ij}^{(\mu B)} = C \varepsilon_L (\mu_i \mathcal{B}_j + \mu_j \mathcal{B}_i) \quad (33)$$

where  $C$  is  $i, j$  independent (complies with the definition in Eq. (12)),  $\varepsilon_L$  is the loop suppression factor and  $\mathcal{B}_i$  corresponds to the product of  $B_i$  and the loop function which also carries the index  $i$ . Together with the tree level contribution  $[m_\nu]_{ij}^{(\mu\mu)} = C \mu_i \mu_j$ , the mass matrix looks like

$$[m_\nu]^{(\mu\mu + \mu B)} = C \begin{pmatrix} \mu_1^2 + 2\varepsilon_L \mathcal{B}_1 \mu_1 & \mu_1 \mu_2 + \varepsilon_L (\mathcal{B}_1 \mu_2 + \mathcal{B}_2 \mu_1) \\ \mu_1 \mu_2 + \varepsilon_L (\mathcal{B}_1 \mu_2 + \mathcal{B}_2 \mu_1) & \mu_2^2 + 2\varepsilon_L \mathcal{B}_2 \mu_2 \end{pmatrix}. \quad (34)$$

After diagonalization, we see that the lighter eigenvalue is suppressed with respect to the heavier one by a factor of the order of  $\varepsilon_L^2$ . This suppression is like  $\mathcal{O}(\varepsilon_L)$  for the  $BB$  loops as in that case  $C$  will dependent on  $i, j$  (see Eq. (28)).

Hence we conclude that although in the flavor basis  $\mu B$  loops seems to have comparable contributions to the  $BB$  loops, in the presence of large tree level contributions they contribute insignificantly to the neutrino masses after diagonalization. We will not elaborate on this diagram any further.

#### 4.5 $\mu\lambda$ AND $\mu\lambda'$ LOOPS

There are contributions to the neutrino masses from loops containing both bilinear ( $\mu_i$ ) and trilinear ( $\lambda$  or  $\lambda'$ )  $\mathcal{R}$  couplings (see Fig. 6). The blob on the external line requires some explanation. The neutrino gets

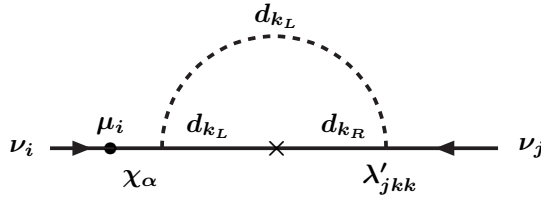


Figure 6: Neutrino Majorana mass generated by  $\mu\lambda'$  loop. There exists another diagram with  $i \leftrightarrow j$ .

converted to a up-type Higgsino through  $\mu_i$  terms. This Higgsino mixes with gauginos which then couple with  $\tilde{d}_L$  and  $d_L$ . The cross on the internal fermion line represents a Dirac mass insertion. Neglecting squark flavor mixing, the  $\mu\lambda'$  contribution to the neutrino mass matrix is given approximately by[14]

$$[m_\nu]_{ij}^{(\mu\lambda')} \approx \sum_k \frac{3}{16\pi^2} g m_{d_k} \frac{\mu_i \lambda'_{jkk} + \mu_j \lambda'_{ikk}}{\tilde{m}}. \quad (35)$$

A factor of 3 appears in the numerator as a color factor. There are similar diagrams comprising of  $\lambda$  instead of  $\lambda'$  with a color factor 1, where the quarks and squarks are replaced by charged leptons and sleptons respectively. If we compare these contributions with  $\lambda'\lambda'$  or  $\lambda\lambda$  type diagrams which suffer from two Yukawa suppressions, we see that this diagram has only one Yukawa suppression. But arguing in the same line as the  $\mu B$  loops, one can see that these diagrams do not contribute to the neutrino masses significantly in the presence of large tree level contributions.

## 5 DISCUSSIONS

In the previous section we have seen that the different contributions to neutrino masses come with different suppression factors. In this section, for completeness, we summarize these contributions in terms of basis independent parameters following Eqs. (10) and comment on their relative contributions.

Approximate expressions of different contributions are given by:

$$\begin{aligned} [m_\nu]_{ij}^{(\mu\mu)} &\sim \delta_i^\mu \delta_j^\mu \cos^2 \beta \tilde{m}, \\ [m_\nu]_{ij}^{(\lambda'\lambda')} &\sim \sum_{l,k} \frac{3}{8\pi^2} \delta_{il}^{\lambda'} \delta_{jk}^{\lambda'} \left( \frac{m_{d_l} m_{d_k}}{\tilde{m}^2} \right) \tilde{m}, \\ [m_\nu]_{ij}^{(BB)} &\sim \frac{g^2}{64\pi^2} \frac{\epsilon_H}{\cos^2 \beta} \delta_i^B \delta_j^B \tilde{m}, \\ [m_\nu]_{ij}^{(\mu B)} &\sim \frac{g^2}{64\pi^2} \frac{\epsilon'_H}{\cos \beta} (\delta_i^\mu \delta_j^B + \delta_j^\mu \delta_i^B) \tilde{m}, \\ [m_\nu]_{ij}^{(\mu\lambda')} &\sim \sum_k \frac{3}{16\pi^2} g (\delta_i^\mu \delta_{jkk}^{\lambda'} + \delta_j^\mu \delta_{ikk}^{\lambda'}) \left( \frac{m_{d_k}}{\tilde{m}} \right) \tilde{m}, \end{aligned} \quad (36)$$

where all the weak scale parameters are taken as  $\tilde{m}$ , which is then factored out as an overall scale.

Although the leading effects are model dependent, some general remarks can be made, if we assume that all the  $\mathcal{R}$  parameters are of the similar order.

- $m_3$  receives a contribution at the tree level and this is the dominant one unless  $\tan \beta$  is very large.  $m_2$  and  $m_1$  are generated at one-loop. Apart from being down by a loop factor  $\sim \frac{1}{16\pi^2}$ , the loops suffer from several other suppression factors.
- Although  $BB$  loop can get a suppression  $\epsilon_H$  from a partial cancellation between different Higgs loops, unless one is too close to the decoupling regime, it is generally likely that this is the most dominant loop contribution. The leading contribution to  $m_2$  comes from this loop. A  $\tan \beta$  dependence comes from  $\epsilon_H$  which partly cancels the  $\cos^2 \beta$  in the denominator reducing the sensitivity to  $\tan \beta$ .

- The trilinear loops  $\lambda'\lambda'$  and  $\lambda\lambda$  suffer from double Yukawa suppression, and contribute very little.
- Apart from a suppression  $\epsilon'_H$  from a partial cancellation between different Higgs loops, the  $\mu B$  loop contribution to the neutrino mass is suppressed in presence of unsuppressed tree level contribution.
- The contributions from  $\mu\lambda$  and  $\mu\lambda'$  loops also have a similar suppression for large tree level contribution and are further down by a single Yukawa suppression.
- Although  $m_1$  can receive contributions from other loops, it can be massive from  $BB$  loops alone if the sneutrinos are non-degenerate. If the amount of non-degeneracy is not small, the dominant contribution to  $m_1$  can very well come from  $BB$  loops and it will be suppressed with respect to  $m_2$  by  $\epsilon_D$ . In this case, the tree level and the  $BB$  loops provide the most important contributions to all neutrinos. Note that since neutrino oscillation data are not sensitive to the overall scale of the neutrino masses, our ignorance of the mechanism that generate  $m_1$  is not problematic.

Now we can comment on how  $R$ -parity violating supersymmetric models confront the experimental observations. We will estimate the approximate sizes required to fit the data, when the tree level contribution dominates over the loops. However, for detailed numerical analysis see Refs. [27, 28].

If we neglect  $m_1$ , we can infer from Eq. (1),

$$\begin{aligned} m_3 &\sim (\delta^\mu)^2 \cos^2 \beta \tilde{m} \sim 10^{-1} eV \\ m_2 &\sim \frac{g^2}{64\pi^2} \frac{1}{\cos^2 \beta} (\delta^B)^2 \tilde{m} \epsilon_H \sim 10^{-2} eV. \end{aligned}$$

Taking  $\cos \beta \sim 1$  and  $\tilde{m} \sim 100$  GeV, this leads to  $\delta^\mu \sim 10^{-6}$  and  $\delta^B \sim 10^{-5}$ . Here the generation indices of the  $\delta$ 's are dropped for simplicity. All these results are highly parameter space dependent, for example in this simple-minded analysis we take  $\epsilon_H \sim 1$ , but as mentioned earlier, it can be quite small. In addition, from the diagonalization of the tree level mass matrix, one can see that one can get large mixing angles. However to reproduce the exact mass-squared differences and the mixing pattern, in particular the smallish  $\theta_{13}$ , some fine-tuning of the relevant parameters is necessary.

It is interesting to note that the required sizes of the  $\delta$ 's are too small, whereas we expect them to be naturally of order one. So it is interesting to explore flavor models which can naturally explain this smallness. We already mentioned that the approximate alignment between  $\mu_\alpha$  and  $v_\alpha$ , required to explain the smallness of the tree level contribution and hence the small  $\delta_i^\mu$ 's, can be realized[21] if we consider, in addition, the presence of some Abelian horizontal symmetry[29]. The use of an Abelian group that distinguishes between different generations has been quite successful in the quark sector. In Ref. [20], an  $U(1)_H$  group has been considered to get naturally small  $R$ -parity violating parameters, which were then used to estimate different contributions to the neutrino masses. However a potential problem with this simple model is to explain a small  $\theta_{13}$  one needs a mild fine-tuning. However, it might be possible to fit the data with less fine-tuning if one goes for an elaborate model, like those with a more complicated flavor symmetry group[31].

## 6 CONCLUSION

We see that  $R$ -parity violating minimal supersymmetric model can naturally accommodate lepton number violation, which can generate Majorana neutrino masses involving weak scale fields. It provides a viable alternative to the see-saw mechanism. These models are particularly useful to reproduce large mixing angles with a hierarchical spectrum, as indicated by experiments. But one should also keep in mind that, in general, to keep the tree level contribution small enough, some fine-tuning is required.

In this  $R$ -parity violating model, the contributions come from both tree and loop level diagrams, which are highly parameter space dependent. So which of them are the most important ones depend on the model concerned. However in a generic scenario, one expects the tree level contributions give mass to the heaviest neutrino. The other two neutrinos get mass at the loop level.  $BB$  loops, as induced by the sneutrino-anti-sneutrino mixing, are expected to be the dominant loop contributions. The trilinear loops turn out to be rather small.

In addition to the constraints from neutrino masses, the  $R$ -parity violating parameters suffer from tremendously small upper bounds imposed by several flavor violating processes[32, 9, 5]. It is interesting to explore theoretical models which can naturally explain this smallness.

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